Math 211-2015S-W3-Friday (Pg 1)

Review

Newton notation $f'(x)$ for derivative of f at x

Leibnitz notation

$$y = f(x)$$

$\frac{dy}{dx}$ for derivative of f at x

Confusing symbolism because it is hard to make sense of "dy" and "dx" on their own. dy was supposed to be a small (infinitessimal) change in y an dx a small change in x, It took hundreds of years before mathematicians made that precise.

Better to just think of $\frac{dy}{dx}$ as one symbol an not attach meaning to dy and dx or d separately, for now,

The top tells you what function is changing the bottom tells you what variable changes

Units $\frac{dy}{dx}$ has units of y / units of x

$y = f(t)$ t is time in hrs
y is position in miles

$\frac{dy}{dx}$ has units $\frac{mi}{hr}$

New

$\boxed{\text{The second derivative}}$

The derivative of a function is itself a function, so we can calculate the derivative of a derivative

This is called the second derivative

Recall that the derivative is written

$$f'(x) \text{ or } \frac{dy}{dx} \quad (\text{if } y = f(x))$$

The second derivative is written $f''(x)$ or

$$\frac{d^2y}{dx^2} \text{ or } \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Again just think of each as one symbol for $f''(x)$

$\frac{d}{dx}$ is the differentiation operator (rule mapping functions to functions)

$\frac{d}{dx}(y)$ is first derivative $\frac{dy}{dx}$

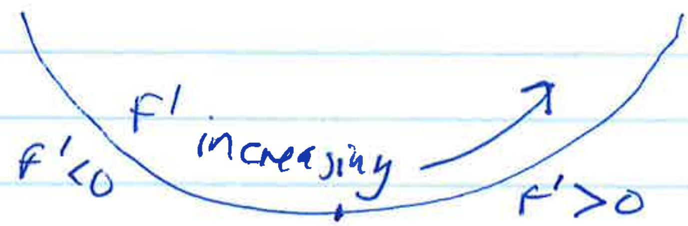
$\frac{d}{dx}\left(\frac{d}{dx}(y)\right)$ is second derivative

$\frac{d^2y}{dx^2}$ Again don't get caught up in notation,

If $f' > 0$ on an interval
then slope is positive on that interval
so f is increasing over that interval

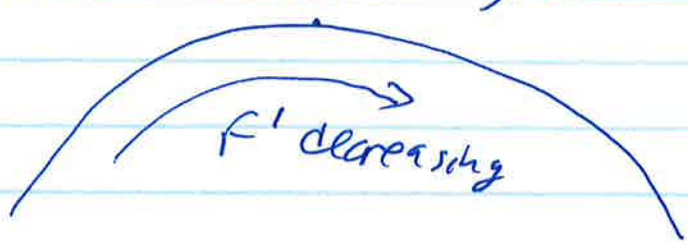
If $f' < 0$ on an interval
then slope is negative on that interval
so f is decreasing on that interval

If $f'' > 0$, on an interval
then f' is increasing on that interval



f is concave up on that interval

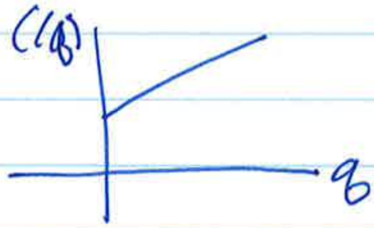
If $f'' < 0$ on an interval
then f' is decreasing on that interval



f is concave down on that interval

Marginal Cost and Revenue

The cost function may be linear

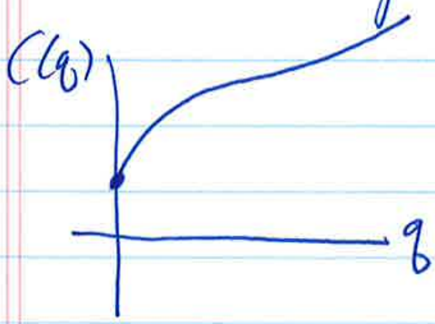


$C(q) = 10 + 3q$

Marginal cost 3 (slope)

Fixed cost 10 (x intercept)

Or it might be nonlinear

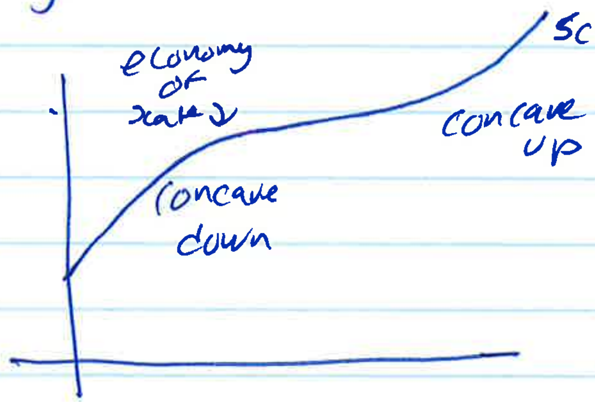


The fixed cost will still be the vertical intercept and the marginal cost will still be slope, (derivative), but now marginal cost depends on q.

$C'(q)$ is marginal cost

$C'(q)$ is approximately the additional cost for one more unit if C' is not changing rapidly near q or equivalently if C'' is not too large near q.

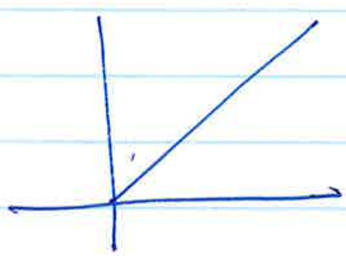
This turns out to be a typical cost function. Why?



economy of scale - production is more efficient at larger quantities

Revenue Function

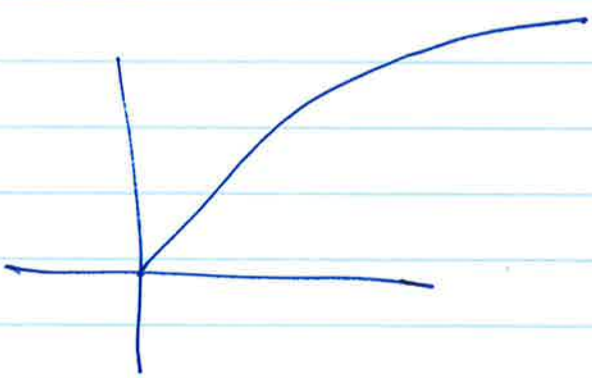
linear:



$R(q) = p \cdot q$ linear

price constant function of quantity

nonlinear:



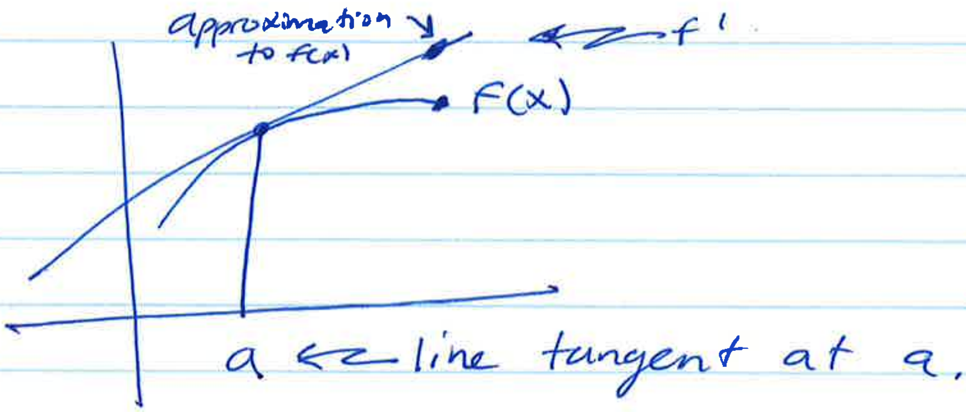
Sometimes you model gluts in the market

(market becomes glutted at high quantity

Math 211-2015 S-W3-Friday (pg 6)

Local linear approximation

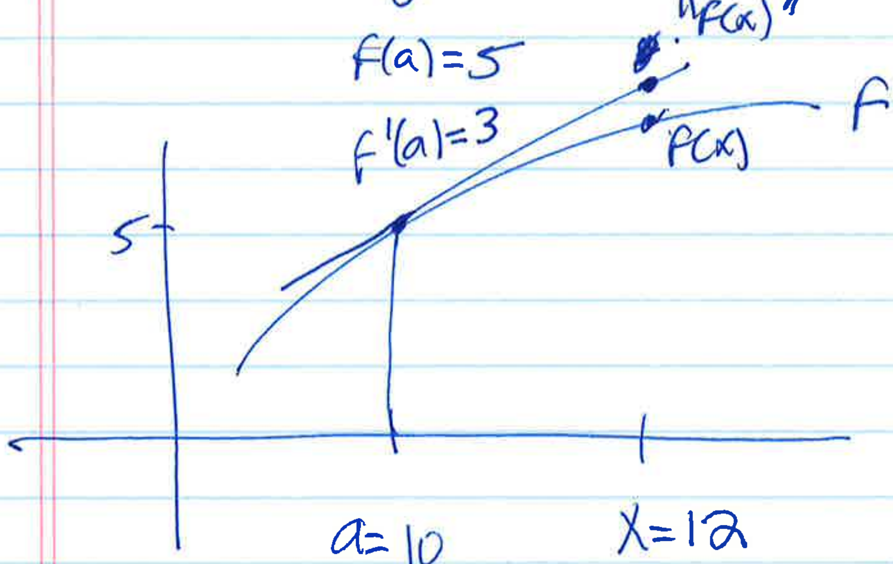
If the second derivative is small enough you can approximate a "curved" nonlinear function with a line



The approximation is good near a and less good far away from a unless the original function is a line

The bigger the concavity i.e. the bigger $|f''|$, the worse the approximation, or rather the closer to a you need to be for the approximation to be good.

How do you find the approximation



You are given f(a), f'(a), a and x

Use difference quotient

$\frac{\text{rise}}{\text{run}} = 3$ $\quad \frac{\text{"}f(x)\text{"} - 5}{12 - 10} = 3$ and solve for "f(x)"

approximation

$y = 11$

$\frac{\text{"}f(x)\text{"} - f(a)}{x - a} = f'(a)$ solve for "f(x)"

approximation

$\text{"}f(x)\text{"} \approx f(a) + f'(a)(x-a)$

The local linear approximation is what is used to show that

The marginal cost is approximately the cost for one more unit

And the marginal revenue is approximately the ~~cost~~ revenue for one more unit

The marginal profit satisfies

$$M\pi = MR - MC$$