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Review

We had two ways of describing the center and spread of a distribution

①  $\bar{x}, s$

if either mean, standard deviation  
are present } sensitive to outliers (both measures)  
not as useful } doesn't show skewness in distribution  
e.g. skewed to left  
or skewed to right

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

sum of obs  
num of obs

$$= \frac{1}{n} \sum x_i$$

Sigma-notation

$$s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

deviations

Squared deviations

Sum of squared deviations

Average of squared deviations dividing by  $n-1$  instead of  $n$

square root

$s^2$  is variance

② min,  $Q_1$ , M,  $Q_3$ , max

a, K, a 5 number summary

minimum, 1<sup>st</sup> Quartile, Median, 3<sup>rd</sup> Quartile, maximum  
0<sup>th</sup> percentile, 25<sup>th</sup> percentile, 50<sup>th</sup> percentile, 75<sup>th</sup> per, 100<sup>th</sup>

resistant to outliers (all measures, except  
min and max)

→ but in a modified box-plot

you exclude outliers (or suspected  
outliers) from min and max  
in that case the min and  
max ~~would~~ as displayed would  
be resistant (to outliers)

Median - point where  
half the obs are above  
half are below

$Q_1$  - Median of lower half of obs

$Q_3$  - Median of upper half of obs

min/max the smallest and largest

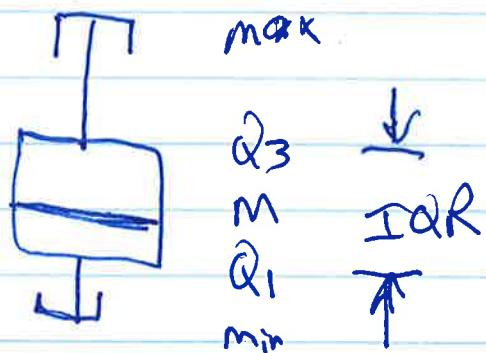
obs (in a modified box plot  
you exclude <sup>suspected outliers</sup> from  
min and max)

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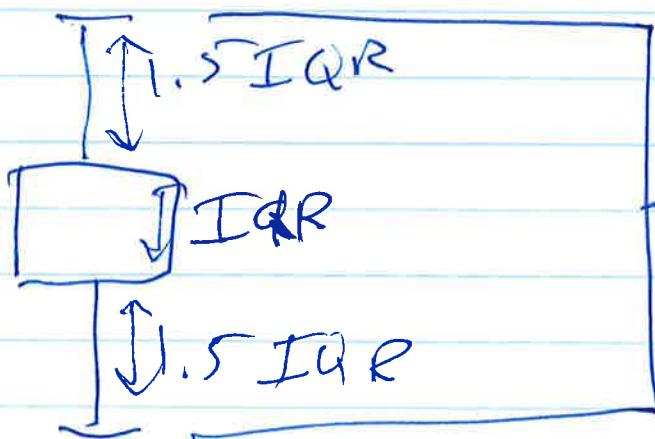
- The five number summary shows skewness of a distribution.

- The five number summary is displayed with a box plot



$Q_3 - Q_1 = \text{IQR}$  = Inner-quartile range.

The IQR is useful for flagging suspected outliers



Fences

beyond which  
we deem  
points suspected  
outliers.

In a modified box plot the plotted min and max are inside fences and obs outside are plotted as asterisks.

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Show homework [4]

New: A transformation is a function that transforms an old variable into a new ~~is~~ variable.

$$\cancel{\text{old variable}} \quad x_{\text{new}} = f(x_{\text{old}})$$

Transformation is just another name for a function.

When we use the word transformation we think of the function as transforming the old variable

### Examples

(a) If ~~is~~  $x_{\text{old}}$  is distance measured in kilometers and  $x_{\text{new}}$  is ~~is~~ distance measured in miles

$$\cancel{\text{new}} = f(\cancel{x})$$

$$x_{\text{new}} = .62 x_{\text{old}}$$

In familiar function notation

$$y = .62x = f(x)$$

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Example 2

$X_{\text{old}}$  = temperature measured in deg F°

$X_{\text{new}}$  = temperature measured in deg C°

~~REMEMBER~~

$$X_{\text{new}} = \frac{5}{9}(X_{\text{old}} - 32)$$

$$y = \frac{5}{9}x - 32$$

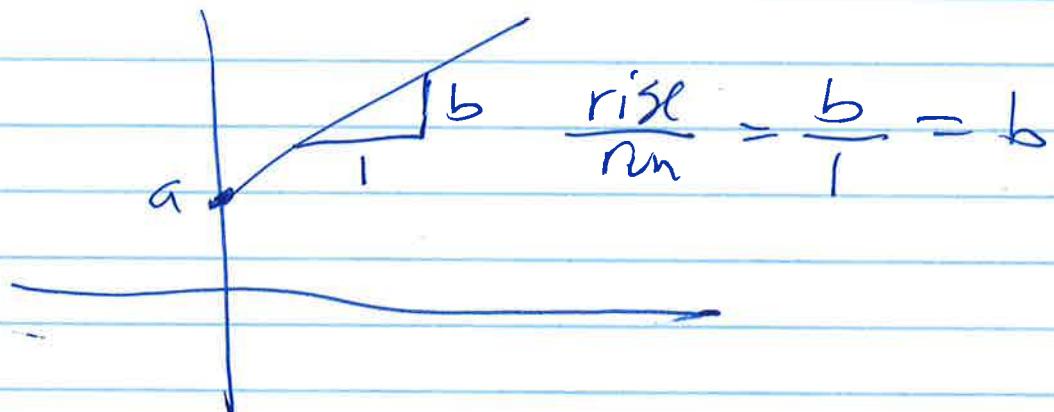
Unit conversions are usually linear functions (graphs are lines)

These are called

linear transformations

$y = mx + b$  in familiar notation

$X_{\text{new}} = a + b X_{\text{old}}$  book's notation

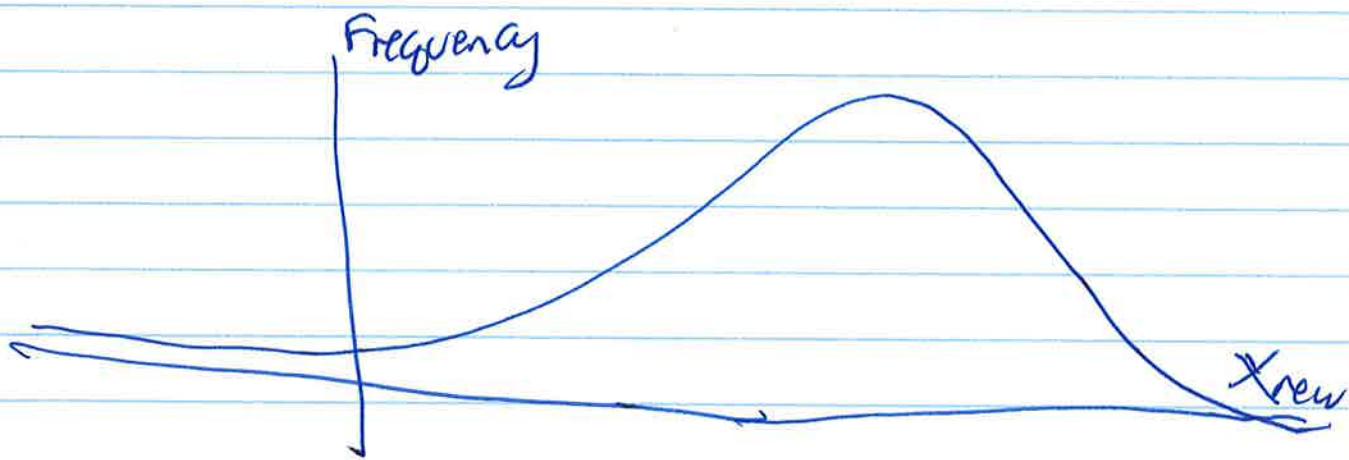
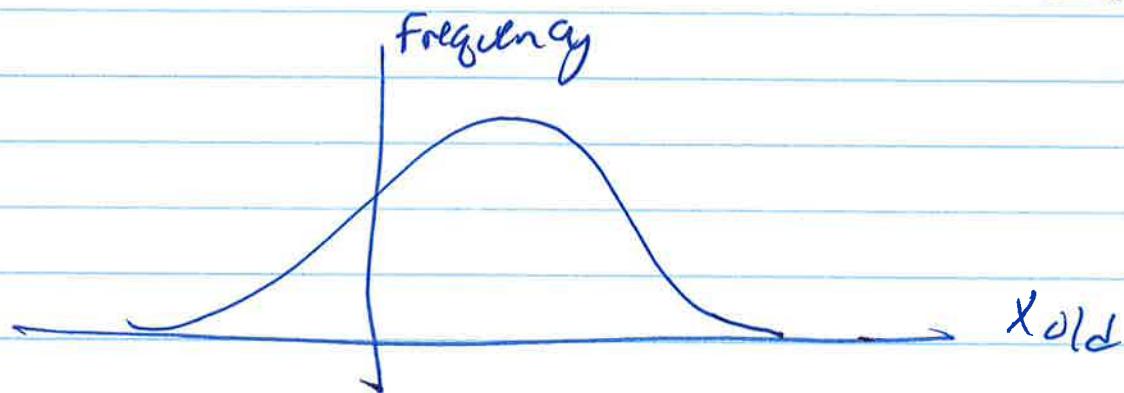


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Linear transformations do not change the shape of a distribution

$X_{\text{old}}$  right skewed  $\Leftrightarrow X_{\text{new}}$  right skewed  
 $X_{\text{old}}$  symmetric unimodal  $\Leftrightarrow X_{\text{new}}$  symmetric unimodal



$$X_{\text{new}} = a + b X_{\text{old}}$$

a shifts histogram

b shrinks histogram if  $|b| < 1$

expands " "

Flips

$|b| > 1$   
if  $b < 0$

peaks, gaps, and skewness remain.

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## Density Curves

A density curve is a smooth approximation to the irregular bars of a histogram.

For a histogram there were three plotting alternatives

- A: Frequency - count of obs in bin
- B: Relative Frequency -  $\text{Freq} / \text{total number of obs}$
- C: Density -  $\text{Relative Freq} / \text{bin width}$

Why pick density? Suppose you collect more and more data. You are likely to see less and less noise in histogram, with A you will get more and more obs in each bin. Therefore, the vertical scale will keep increasing. Not so with B and C.

Another thing to do is to decrease the width of bins as you increase the number of data points. This will give a smoother and smoother histogram.

B will change (vertical scale) as you decrease bin width  
C won't.

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That's why we like C.

As we approach a histogram  
with more and more data and  
smaller and smaller bin width  
we approach a histograms

Show with stat crunch

For A (Frequency) the sum of bin  
heights is  $n$

For B ~~probability~~ the sum of bin heights is 1

For C, the area of the bins is 1

For density curves, like for C.

(1) the area under curve is 1

(2) the curve is always on or above  
the horizontal axis,

A density wrve is any curve that satisfies  
(1) and (2); A distribution is completely  
described by its density curve.

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So instead of giving the 5-number summary of a variable it would be better to give the density curve.

Problem though: You can never have enough data to be sure you know the density curve.

There are ways to estimate density curves from data. But StatCrunch does not do this

What StatCrunch will do instead is overlay your histogram of data with a density curve of a standard distribution

There are infinitely many distributions (any density curve determines one)

Only a few have names (the standard distributions)

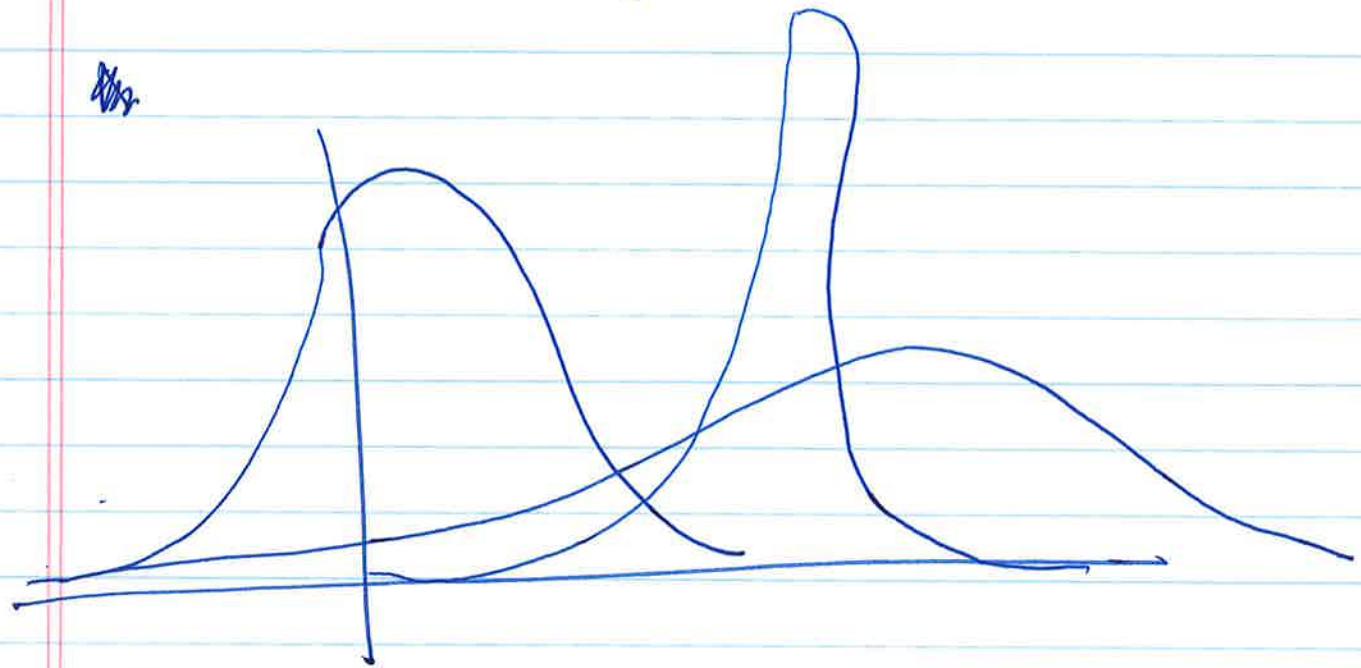
The most important standard distribution is the Normal Distribution

Its density curve is the bell curve.

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Bell Curves are determined by  
their mean and standard deviation



Mean of data written  $\bar{x}$

Standard deviation of data written  $s$

Mean of density curve written  $\mu$

Std dev  $\sigma$

We'll give precise defns next week