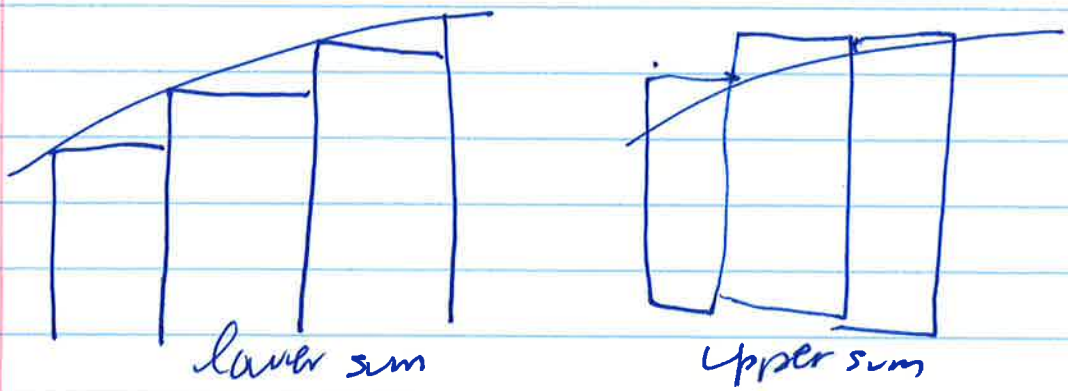
Math 21 - 2015S - W8 - Wednesday

Review

We talked about estimating the area under a curve with a bunch of rectangles. The estimate was the sum of the areas of the rectangles



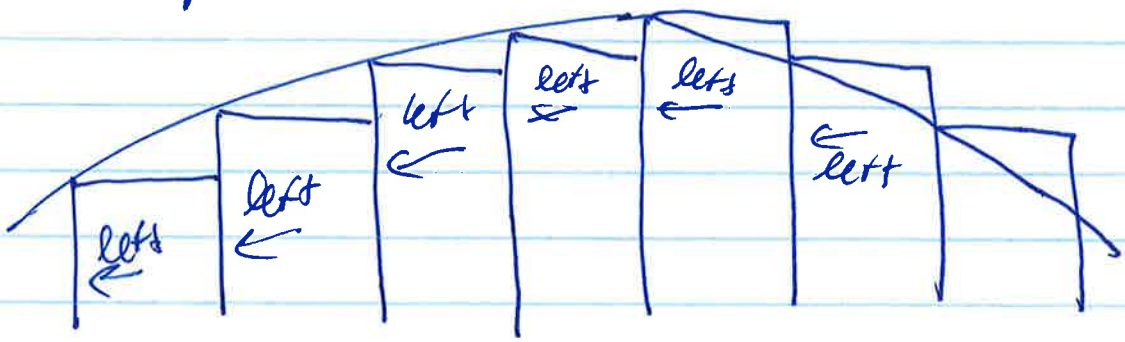
We talked about a lower sum and an upper sum. For the lower sum we made the height of each rectangle the lowest point of the function on the interval spanned by the rectangle. And the upper sum ~~was~~ the highest point was used.

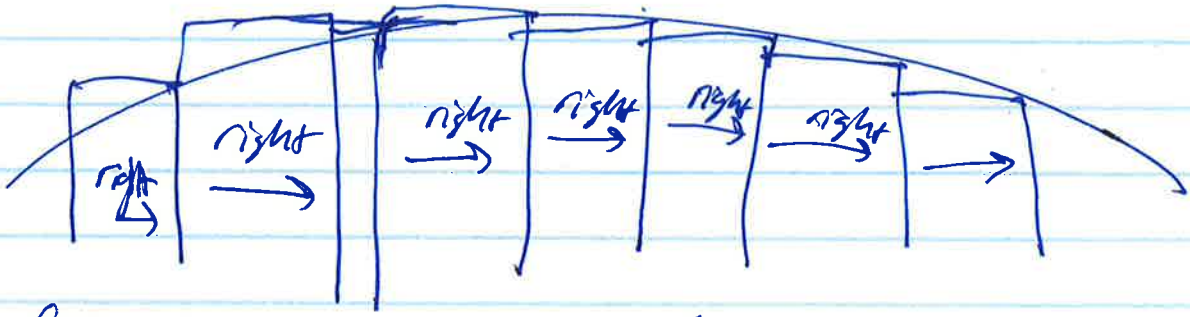
The lower sum gave you a lower estimate. Upper sum. Upper estimate.

The two together gave bounds on Area under curve.

Sampling more frequently gave tighter bounds.

New There is a related notion of left-handed sum and right-handed sum





rectangles left-handed sum and lower sum coincide where function is increasing

left- " " upper

" decreasing

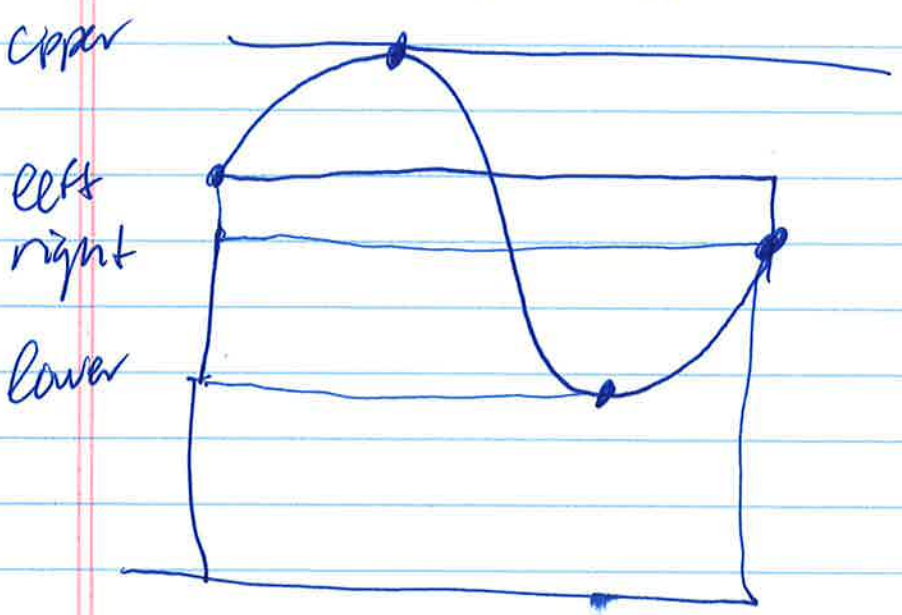
right-handed upper

increasing

~~left~~ right-hand " lower

decreasing

On an interval like this



all 4 are different

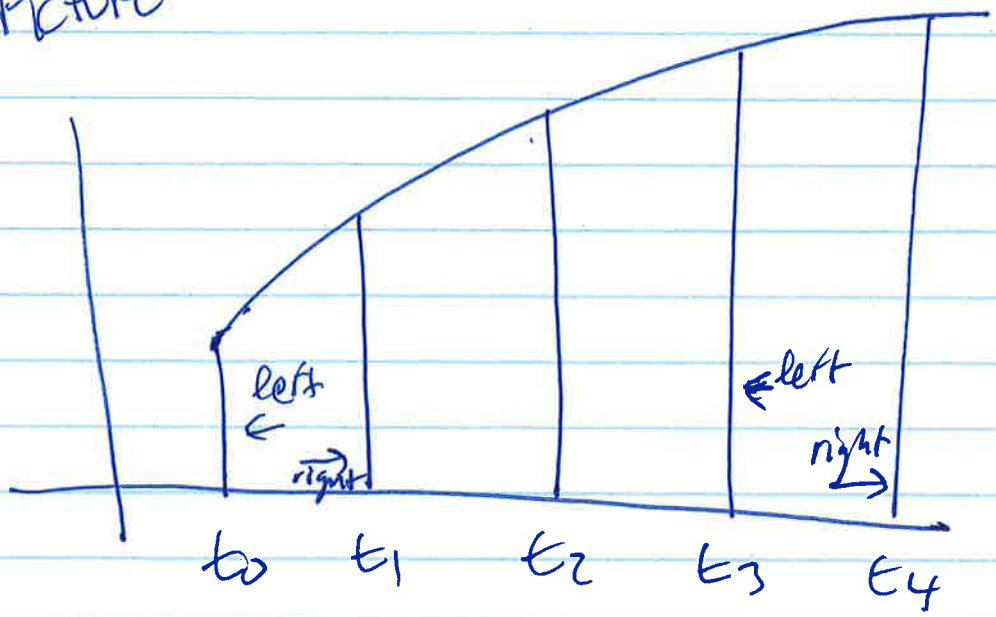
It turns out it is easier to write a formula for a left and a right handed sum

n is number of rectangles
Δt is width of rectangles

Right-hand sum $\sum_{i=1}^{n} f(t_i)\,\Delta t$

Left hand sum $\sum_{i=0}^{n-1} f(t_i)\,\Delta t$

Picture



The definite integral of F from a to b is the area under the graph between a and b
(if F is positive $\xrightarrow{\text{save}}$ F negative for later)

The integral is defined precisely as the limit as n increases to infinity and Δt decreases to 0 of

- RHS
- LHS
- US
- LS

These things should all coincide if they don't integral doesn't exists, For our purposes integral will always exist

Called Riemann Sums

The integral is sometimes called the Riemann integral. There are other kinds in other courses

The integral is written

$$\int_a^b f(t)\,dt$$

here f is called the integrand

a and b are called the limits of integration

Like Leibniz notation just think of the notation as one symbol — don't get caught up with trying to understand what each part means eg dt

§5.3 The definite integral as area

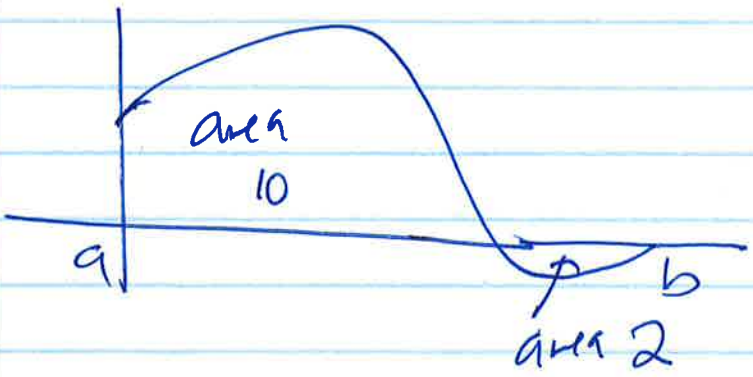
I said the integral is area under the curve when f is positive

If f is negative in some places and positive in others

then the area where f is positive add to integral

and the area where f is negative subtracts from integral

thus



whole integral is

$$\int_a^b f(t)\,dt = 10 - 2 = 8$$