Homework #17
Stat 202

**4.76 Mean of the distributions of errors.** Typographical and spelling errors can be either "nonword errors" or "word errors." A nonword error is not a real word, as when "the" is typed as "teh." A word error is a real word, but not the right word, as when "lose" is typed as "loose." When undergraduates are asked to write a 250-word essay (without spell-checking), the number of nonword errors has the following distribution:

| Errors | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Probability | 0.1 | 0.3 | 0.3 | 0.2 | 0.1 |

The number of word errors has this distribution:

| Errors | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Probability | 0.4 | 0.3 | 0.2 | 0.1 |

What are the mean numbers of nonword errors and word errors in an essay?

**4.82 The effect of correlation.** Find the mean and standard deviation of the total number of errors (nonword errors plus word errors) in an essay if the error counts have the distributions given in Exercise 4.76 and

**(a)** the counts of nonword and word errors are independent.

**(b)** students who make many nonword errors also tend to make many word errors, so that the correlation between the two error counts is 0.5.

**4.83 Means and variances of sums.** The rules for means and variances allow you to find the mean and variance of a sum of random variables without first finding the distribution of the sum, which is usually much harder to do.

**(a)** A single toss of a balanced coin has either 0 or 1 head, each with probability 1/2. What are the mean and standard deviation of the number of heads?

**(b)** Toss a coin four times. Use the rules for means and variances to find the mean and standard deviation of the total number of heads.

**(c)** Example 4.23 (page 251) finds the distribution of the number of heads in four tosses. Find the mean and standard deviation from this distribution. Your results in parts (b) and (c) should agree.

**4.84 CHALLENGE Toss a four-sided die twice.** Role-playing games like Dungeons & Dragons use many different types of dice. Suppose that a four-sided die has faces marked 1, 2, 3, 4. The intelligence of a character is determined by rolling this die twice and adding 1 to the sum of the spots. The faces are equally likely and the two rolls are independent. What is the average (mean) intelligence for such characters? How spread out are their intelligences, as measured by the standard deviation of the distribution?

**4.76.** The means are

$$(0)(0.1) + (1)(0.3) + (2)(0.3) + (3)(0.2) + (4)(0.1) = 1.9 \text{ nonword errors and}$$
$$(0)(0.4) + (1)(0.3) + (2)(0.2) + (3)(0.1) = \quad 1 \text{ word error}$$

**4.82.** Let $N$ and $W$ be nonword and word error counts. In Exercise 4.76, we found $\mu_N = 1.9$ errors and $\mu_W = 1$ error. The variances of these distributions are $\sigma_N^2 = 1.29$ and $\sigma_W^2 = 1$, so the standard deviations are $\sigma_N \doteq 1.1358$ errors and $\sigma_W = 1$ error. The mean total error count is $\mu_N + \mu_W = 2.9$ errors for both cases. **(a)** If error counts are independent (so that $\rho = 0$), $\sigma_{N+W}^2 = \sigma_N^2 + \sigma_W^2 = 2.29$ and $\sigma_{N+W} \doteq 1.5133$ errors. (Note that we add the *variances,* not the standard deviations.) **(b)** With $\rho = 0.5$, $\sigma_{N+W}^2 = \sigma_N^2 + \sigma_W^2 + 2\rho\sigma_N\sigma_W \doteq 2.29 + 1.1358 = 3.4258$ and $\sigma_{N+W} \doteq 1.8509$ errors.

**4.83.** **(a)** The mean for one coin is $\mu_1 = (0)\left(\frac{1}{2}\right) + (1)\left(\frac{1}{2}\right) = 0.5$ and the variance is $\sigma_1^2 = (0 - 0.5)^2\left(\frac{1}{2}\right) + (1 - 0.5)^2\left(\frac{1}{2}\right) = 0.25$, so the standard deviation is $\sigma_1 = 0.5$. **(b)** Multiply $\mu_1$ and $\sigma_1^2$ by 4: $\mu_4 = 4\mu_1 = 2$ and $\sigma_4^2 = 4\sigma_1^4 = 1$, so $\sigma_4 = 1$. **(c)** Note that because of the symmetry of the distribution, we do not need to compute the mean to see that $\mu_4 = 2$; this is the obvious balance point of the probability histogram in Figure 4.7. The details of the two computations are

$$\mu_W = (0)(0.0625) + (1)(0.25) + (2)(0.375) + (3)(0.25) + (4)(0.0625) = 2$$
$$\sigma_W^2 = (0-2)^2(0.0625) + (1-2)^2(0.25) + (2-2)^2(0.375) + (3-2)^2(0.25) + (4-2)^2(0.0625) = 1.$$

**4.84.** If $D$ is the result of rolling a single four-sided die, then $\mu_D = (1+2+3+4)\left(\frac{1}{4}\right) = 2.5$, and $\sigma_D^2 = [(1-2.5)^2 + (2-2.5)^2 + (3-2.5)^2 + (4-2.5)^2]\frac{1}{4} = 1.25$. Then for the sum

$I = D_1 + D_2 + 1$, we have mean intelligence $\mu_I = 2\mu_D + 1 = 6$. The variance of $I$ is $\sigma_I^2 = 2\sigma_D^2 = 2.5$, so $\sigma_I \doteq 1.5811$.