

Confidence Intervals

From a sample

We can't infer the population mean μ .

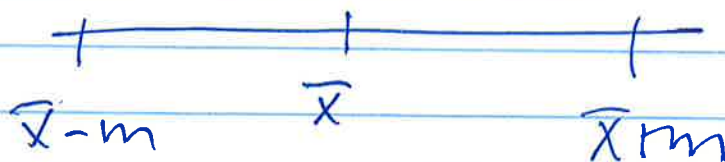
The best we can do is \bar{x}

But we can write down an interval which contains μ 95% of the time

The interval has the form

$$\bar{x} \pm m$$

↑ estimate ↑ margin of error



$$m = z^* \frac{\sigma}{\sqrt{n}}$$

where z^* is about 2
for 95% confidence
intervals

If you use software instead of
68-95-99.7 Rule you'll find

$z^* = 1.960$ for 95% confidence intervals

by assumption

Pg 2

Let's say $\bar{x} = 10$, $\sigma = 2$, $n = 16$
Find 95% confidence interval, use $z^* = 2$

$$\begin{aligned}\bar{x} \pm z^* \sigma / \sqrt{n} &= 10 \pm 2 \cdot 2 / 4 \\ &= 10 \pm 1 \\ &= (9, 11)\end{aligned}$$

what is
margin of error?

Let's say we draw another sample
with $n = 16$, we have new \bar{x}
 10.5 , $\sigma = 2$ (same assumption) $n = 16$

$$\bar{x} \pm z^* \sigma / \sqrt{n}$$

$$10.5 \pm 2 \cdot 2 / \sqrt{16} = (9.5, 11.5)$$

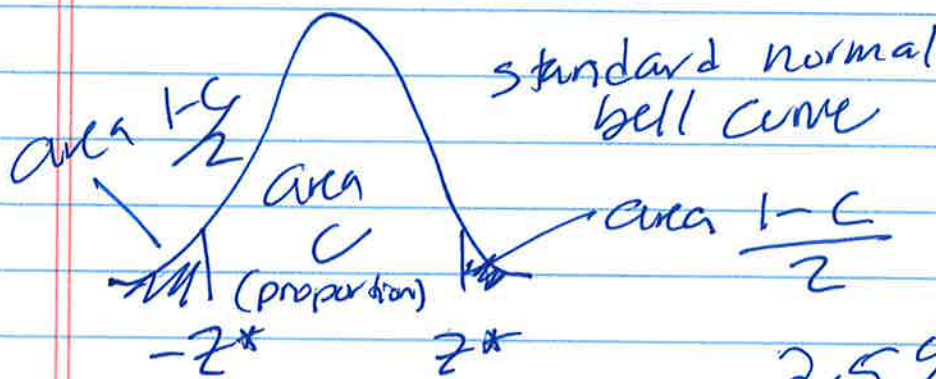
What is margin of error?

It is the same for all samples of size 16
under our assumption $\sigma = 2$ and using
95% confidence intervals so $z = 2$.

Every time we draw a new sample
we are going to get a new \bar{x} thus
a new interval. But we are guaranteed that
in a large number of samples each with n
95% will contain true value of parameter μ .

Confidence intervals are traditionally 95% but other confidence levels can be considered

Confidence level	z^*
68%	1
95%	2
99.7%	3



Here C is expressed as a proportion

$$95\% = .95$$

$$2.5\% = 0.025$$

You can find z with StatCrunch

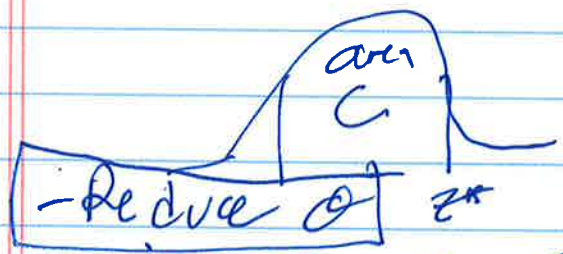
$$z^* = 1.95994$$

SUPPOSE that you calculate a margin of error and decide that it is too large

$$m = z^* \sigma / \sqrt{n}$$

What can you do

- Use a lower level of confidence
 z^* will be smaller for lower confidence



- increase n Sample size (best option)

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+ Increasing n by factor of 4 decreases m by factor of 2

+ Increasing n by factor of 100 decreases m by factor of 10

What sample size do you need to have a specified margin of error.

$$m = z * \sigma / \sqrt{n}$$

Solve for n

$$n = \left(\frac{z * \sigma}{m} \right)^2$$