Stat 202
Spring 2015
Exam 2
3/27/14
Time Limit: 75 Minutes

Name (Print): Key

This exam contains 6 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. A calculator is OK. You may use the browser on your computer to access StatCrunch. You may not visit any other websites. You will not need to download any data from my website or anywhere else. You may use a calculator app on your computer, but no other computer use is allowed.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work,** in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- **Write down what you input to StatCrunch, otherwise you won't get partial credit for a wrong answer.**
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	20	
2	10	
3	30	
4	20	
5	10	
6	10	
Total:	100	

100	90s	80s	70s	60s	50s
0	14	9	1	1	2

Formulas that may or may not be useful:

- If X is a discrete random variable that takes on values $x_1, x_2, \ldots x_n$ with respective probabilities $p_1, p_2, \ldots, p_n$, then its mean is given by:

$$\mu_X = \sum x_i p_i$$

- The mean of a linear transformation of a random variable X is given by the following (where a and b are numbers, not random variables):

$$\mu_{a+bX} = a + b\mu_X$$

- The mean of a sum of two random variables X and Y is given by:

$$\mu_{X+Y} = \mu_X + \mu_Y$$

- A more general formula that combines the two above; here $X_1, \ldots, X_n$ are random variables and a and $b_1, \ldots, b_n$ are numbers:

$$\mu_{a+b_1X_1+b_2X_2+\ldots b_nX_n} = a + b_1\mu_{X_1} + b_2\mu_{X_2} + \ldots b_n\mu_{X_n}$$

- If X is a discrete random variable that takes on values $x_1, x_2, \ldots x_n$ with respective probabilities $p_1, p_2, \ldots, p_n$, then its variance is given by:

$$\sigma_X^2 = \sum (x_i - \mu)^2 p_i$$

- The variance of a linear transformation of a random variable X is given by the following (where a and b are numbers, not random variables):

$$\sigma_{a+bX}^2 = b^2\sigma_X^2$$

- The mean of a sum of two independent random variables X and Y is given by:

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$$

- A more general formula that combines the two above; here $X_1, \ldots, X_n$ are independent random variables and a and $b_1, \ldots, b_n$ are numbers:

$$\sigma_{a+b_1X_1+b_2X_2+\ldots b_nX_n}^2 = b_1^2\sigma_{X_1}^2 + b_2^2\sigma_{X_2}^2 + \ldots b_n^2\sigma_{X_n}^2$$

- Central limit theorem: For a population with mean μ and standard deviation σ, and for samples chosen of size n, the distribution of the sample mean is approximately normal:

$$\bar{x}_n \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

1. (20 points) The results for the first two (hypothetical) statistics exams are in!

Name	Amy	Joe	Sue	Jan	Dan	Eva	Mia
1^{st} exam, S_1	99	83	68	85	73	78	97
2^{nd} exam, S_2	96	91	73	88	72	80	90

(a) (10 points) Find the correlation between S_1 and S_2.

Stat → Summary Stats → Correlation
pick the two columns
Correlation is ~~[illegible]~~ 0.9045

(b) (10 points) Write the equation for the regression line, showing one score as a function of the other. Use the first score as the explanatory variable and the second score as the response variable.

Stat → Regression → Simple Linear
X variable : score 1 column
Y variable : score 2 column

$(\text{Score } 2) = \overset{23.4}{\cancel{[illegible]}} + \overset{0.7306}{\cancel{[illegible]}}(\text{Score } 1)$

2. (10 points) (a) (5 points) What is the correlation between random variables X and Y where the two are related by $Y = -0.5X + 10$?

perfect linear relationship, negative slope
Correlation = −1

(b) (5 points) What is the correlation between random variables X and Y assuming X and Y are independent (knowing the value of one doesn't tell you anything about the value of the other).

Correlation = 0

3. (30 points) A loaded four-sided die has sides labeled by color: blue, red, yellow and green. The probability of rolling a blue is 0.1. The probability of rolling a red is 0.3. The probability of rolling a yellow is 0.5. If blue is rolled you lose \$100. If red is rolled you win \$10. If yellow is rolled you win \$50, and if green is rolled you win \$30.

   (a) (10 points) What is the probability of rolling a green?

blue .1
red .3
yellow .5
+ green ?
1

$.1 + .3 + .5 + ? = 1.0$

probability of rolling a green is .1

   (b) (10 points) What is the mean of the amount of money you win by playing the game once?

money won = X

	blue	red	yellow	green
Values of X	−100	+10	+50	30
probability	.1	.3	.5	.1

μ_X = mean money won = $(-100)(.1) + (10)(.3) + (50)(.5) + (30)(.1)$

$= -10 + 3 + 25 + 3 = \boxed{\$21}$

   (c) (10 points) What is the standard deviation of the amount of money you win by playing the game once?

$\sigma_X^2 = \sum (X_i - \mu_X)^2 p_i$

$= (-100-21)^2(.1) + (10-21)^2(.3)$

$+ (50-21)^2(.5) + (30-21)^2(.1)$

$= 1929$

$\sigma_X = \sqrt{1993} = 43.92$

4. (20 points) In a particular game of chance, the mean amount of money the house wins per game is \$10 and the standard deviation is \$5. Let Z be the amount of money won by the house in 100 games. Let W be the average amount of money won by the house per game over these 100 games. You may assume that the results of the games are independent.

(a) (5 points) What is the mean of Z? $X_i =$ ~~number~~ money won in game i

$\mu_Z = \mu_{X_1} + \dots + \mu_{X_{100}}$

$= 100 \cdot 10$

$= 1000$

(b) (5 points) What is the standard deviation of Z? $\sigma^2_{X_i} = 25$

$\sigma_Z^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2 + \dots + \sigma_{X_{100}}^2 = 100 \cdot 25$

$= 2500$

$\sigma_Z = \sqrt{2500} = 50$

(c) (5 points) What is the mean of W?

10

(d) (5 points) What is the standard deviation of W?

$\frac{5}{\sqrt{100}} = \frac{1}{2} = .5$

5. (10 points) A casino is considering how much to charge to play a game that pays the player \$5 with probability 0.2, \$50 with probability 0.02, \$500 with probability 0.002, and otherwise pays nothing. What is the minimum the casino should charge to expect to break even (on average, pay out equal to what they charge)

pays \$5 50 500 0

.2 .02 .002 rest

Mean Amount paid per game is

$= 0 \cdot (\text{rest}) + .2 \cdot 5 + .02 \cdot 50 + .002 \cdot 500$

$= 3$

Casino should charge at least \$3 per game.

6. (10 points) Joe's lucky numbers are 3, 4 and 7. Joe rolls a 12 sided die (with faces numbered 1 through 12) 10 times. If he gets a lucky number on 5 or more rolls, he wins. If he doesn't he loses. What is the probability that Joe wins?

$p = \frac{3}{12} = \frac{1}{4}$ $n = 10$

$P(X \geq 5) =$ ________

Stat → Calculators → ~~Normal~~ Binomial

.0781