

Review

A probability model is a description of a random phenomenon in the language of mathematics

A probability model consists of

- 1) the set of possible outcomes of the phenomenon (sample space)
- 2) the probability of each outcome, ~~more precisely~~ each event (subset of sample space) (we'll see ^{today} why probabilities are assigned to events not outcomes)

Set Theory

A set is a collection of objects

Eg $\{\text{Head, Tails}\}$

$\{\text{People in this room}\}$

$\{\text{Numbers between 1 and 6 (e.g. die faces)}\}$

~~III~~NotionsIs an element of E Is not an element of $\neq$ Is a subset of C Is not a subset of $\neq$

e.g.

give examples

If A and B are sets

A is a subset of B , $A \subset B$, if every element of A is also an element of B

Empty set $\emptyset$, set of no elements,

$\emptyset \subset A$ for every set A
 $A \subset A$ for every set A

The sample space S of a random phenomenon is the set of all possible outcomes of the phenomenon

$\{\text{Heads, Tails}\}$ Coin
 $\{1, 2, 3, 4, 5, 6\}$ Die

An event is a subset of sample space

$\{H\}$, $\{T\}$, $\{H, T\}$, $\emptyset$

$A \cup B$ (A or B)

Venn Diagrams



Elements of $A \cup B$

are in A or in B or Both

$A \cap B$

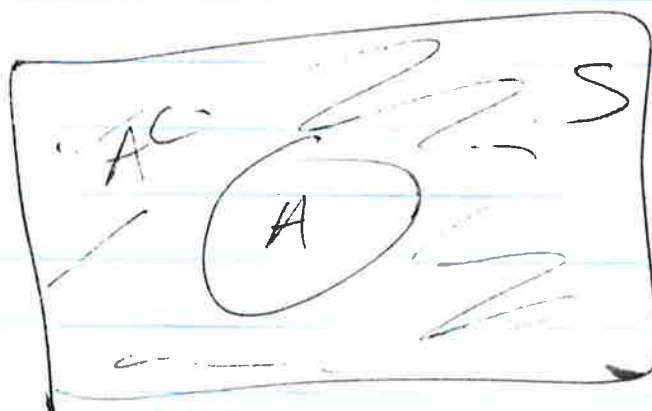
(A and B)



$A \cap B$

Elements of $A \cap B$ are in both A and B

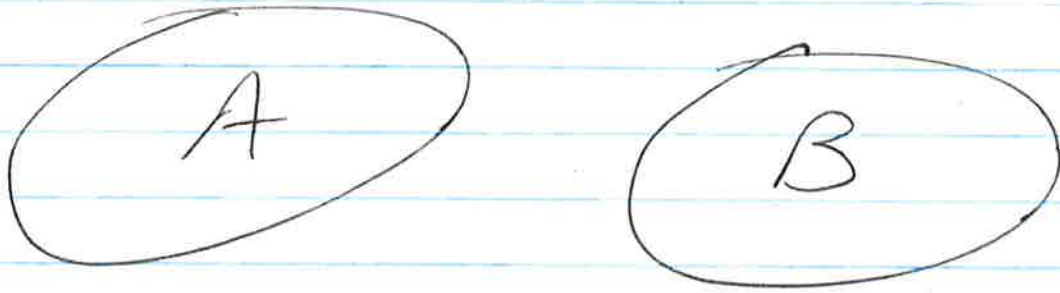
A^c



Elements of A^c are in the sample space S but not in A.

Disjoint sets

HAVE no elements in common



A Probability is a number assigned to an event.

When probabilities are assigned to all events, they must obey certain rules

Rule 1: Each probability must be a number between 0 and 1

For all events A , $0 \leq P(A) \leq 1$

Rule 2: All possible outcomes together have probability 1

$$P(S) = 1$$

↑ sample space

Rule 3: If two events have no outcomes in common, the probability that either one occurs or the other is the sum of the individual probabilities

If A and B are disjoint events
 $P(A \cup B) = P(A) + P(B)$

Rule 4: The probability that an event does not occur is one minus the probability that it does ~~occur~~ occur.

For all events A , $P(A^c) = 1 - P(A)$

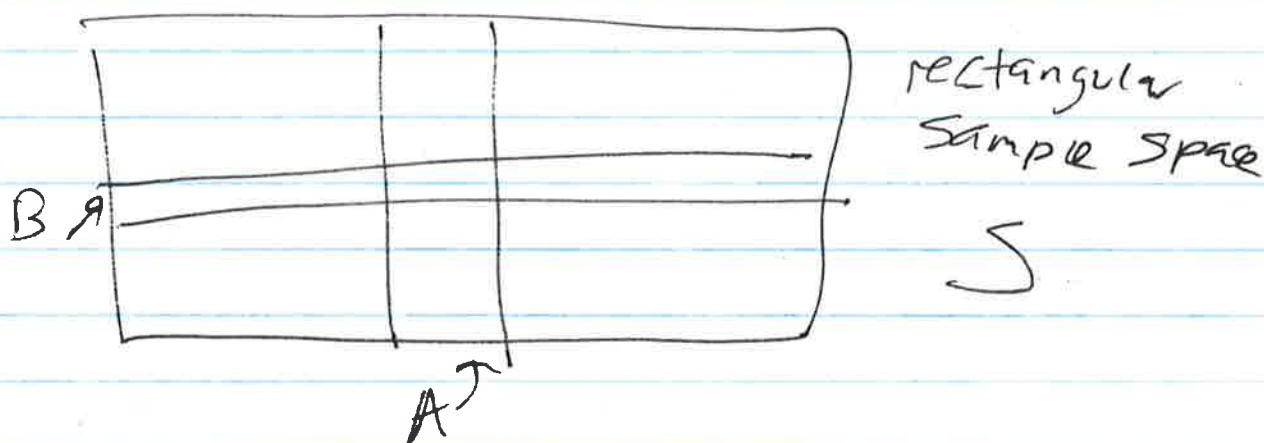
Rule 5: For Rule 5 we must define independent events

Two events A and B are independent if knowing that one occurs does not change the probability that the other occurs (successive coin tosses, sex of children)

First heads can't predict if second heads or tails

First born a boy doesn't tell you if second is a boy or girl.

The picture that goes with this definition is



Areas of sets are there probabilities

A is a vertical bar B is a horizontal bar

$$P(A \cap B) = P(A)P(B)$$

↑ area of intersection ↑ base × height

Rule 5: IF A and B are independent
 $P(A \cap B) = P(A)P(B)$

If all outcomes are equally likely then

Probability (each outcome)

$$= \frac{1}{\text{total number of outcomes}}$$

And if A is an event in a probability model where all outcomes are equally likely

Then

$$P(A) = \frac{\text{total number of outcomes in } A}{\text{total number of outcomes in } S}$$

↗
Entire
Sample
Space

§4.3 Random Variables

A random variable is a function that assigns a number to each outcome of a random phenomenon

Sample
Space

TTT $\rightarrow$ 0

HTT $\rightarrow$ 1

THT $\rightarrow$ 1

TTH $\rightarrow$ 1

HHT $\rightarrow$ 2

HTH $\rightarrow$ 2

TTH $\rightarrow$ 2

HHH $\rightarrow$ 3

~~~~~

outcomes  $\leftarrow$  these are numbers assigned to outcomes

EG: Flip a coin  
three times in a  
row.

In this case the number assigned happens to be the number of heads seen. in 3 flips

However any assignment of numbers to outcomes is a random variable,

Not all of these assignments (RVs) would be useful.