Homework #18
Stat 202

5.9 Generating a sampling distribution. Let's illustrate the idea of a sampling distribution in the case of a very small sample from a very small population. The population is the 10 scholarship players currently on your men's basketball team. For convenience, the 10 players have been labeled with the integers 0 to 9. For each player, the total amount of time spent (in minutes) on Facebook during the last month is recorded in the table below.

Player	0	1	2	3	4	5	6	7	8	9
Total Time (min)	370	290	358	366	323	319	358	309	327	368

The parameter of interest is the average amount of time on Facebook. The sample is an SRS of size $n = 3$ drawn from this population of players. Because the players are labeled 0 to 9, a single random digit from Table B chooses one player for the sample.

(a) Find the mean of the 10 players in the population. This is the population mean μ.

(b) Use Table B to draw an SRS of size 3 from this population (Note: you may sample the same player's time more than once). Write down the three times in your sample and calculate the sample mean $\bar{x}$. This statistic is an estimate of μ.

(c) Repeat this process 10 times using different parts of Table B. Make a histogram of the 10 values of $\bar{x}$. You are constructing the sampling distribution of $\bar{x}$.

(d) Is the center of your histogram close to μ? Would it get closer to μ the more times you repeated this sampling process? Explain.

5.10 Total sleep time of college students. In Example 5.1, the total sleep time per night among college students was approximately Normally distributed with mean $\mu = 7.02$ hours and standard deviation $\sigma = 1.15$ hours. Suppose you plan to take an SRS of size $n = 200$ and compute the average total sleep time.

(a) What is the standard deviation for the average time?

(b) Use the 95 part of the 68–95–99.7 rule to describe the variability of this sample mean.

(c) What is the probability that your average will be below 6.9 hours?

5.11 Determining sample size. Recall the previous exercise. Suppose you want to use a sample size such that about 95% of the averages fall within ±5 minutes of the true mean $\mu = 7.02$.

(a) Based on your answer to part (b) in Exercise 5.8, should the sample size be larger or smaller than 200? Explain.

(b) What standard deviation of the average do you need such that about 95% of all samples will have a mean within 5 minutes of μ?

(c) Using the standard deviation calculated in part (b), determine the number of students you need to sample.

5.12 Songs on an iPod. An iPod has about 10,000 songs. The distribution of the play time for these songs is highly skewed. Assume that the standard deviation for the population is 280 seconds.

(a) What is the standard deviation of the average time when you take an SRS of 10 songs from this population?

(b) How many songs would you need to sample if you wanted the standard deviation of $\bar{x}$ to be 15 seconds?

5.13 Bottling an energy drink. A bottling company uses a filling machine to fill cans with an energy drink. The cans are supposed to contain 250 milliliters (ml). The machine, however, has some variability, so the standard deviation of the size is $\sigma = 3$ ml. A sample of 6 cans is inspected each hour for process control purposes, and records are kept of the sample mean volume. If the process mean is exactly equal to the target value, what will be the mean and standard deviation of the numbers recorded?

5.14 Play times for songs on an iPod. Averages of several measurements are less variable than individual measurements. Suppose the true mean duration of the play time for the songs in the iPod of Exercise 5.12 is 350 seconds.

(a) Assuming the play times to be Normally distributed, sketch on the same graph the two Normal curves, one for sampling a single song and one for the mean of 10 songs.

(b) What is the probability that the sample mean differs from the population mean by more than 19 seconds when only 1 song is sampled?

(c) How does the probability that you calculated in part (b) change for the mean of an SRS of 10 songs?

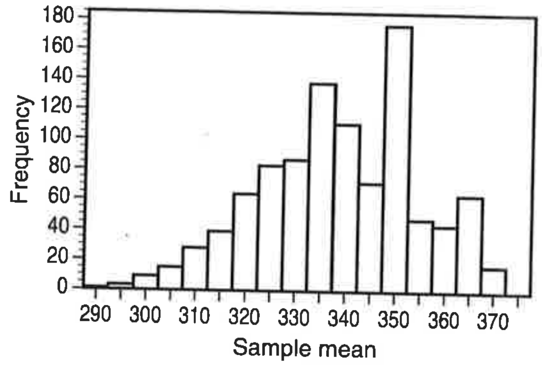
5.16 Number of friends on Facebook. Facebook provides a variety of statistics on their Web site that detail the growth and popularity of the site.[4] One such statistic is that the average user has 130 friends. This distribution only takes integer values, so it is certainly not Normal. We'll also assume it is skewed to the right with a standard deviation $\sigma = 85$. Consider an SRS of 30 Facebook users.

(a) What are the mean and standard deviation of the total number of friends in this sample?

(b) What are the mean and standard deviation of the mean number of friends per user?

(c) Use the central limit theorem to find the probability that the average number of friends in 30 Facebook users is greater than 140.

5.9. **(a)** $\mu = 3388/10 = 338.8$. **(b)** The scores will vary depending on the starting row. The smallest and largest possible means are 290 and 370. **(c)** Answers will vary. Shown on the right is a histogram of the (exact) sampling distribution. With a sample size of only 3, the distribution is noticeably non-Normal. **(d)** The center of the exact sampling distribution is μ, but with only 10 values of $\bar{x}$, this might not be true for student histograms.



Note: *This histograms were found by considering all 1000 possible samples.*

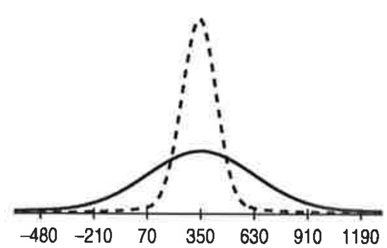
5.10. **(a)** $\sigma_{\bar{x}} = \sigma/\sqrt{200} \doteq 0.08132$. **(b)** With $n = 200$, $\bar{x}$ will be within ± 0.16 (about 10 minutes) of $\mu = 7.02$ hours. **(c)** $P(\bar{x} \le 6.9) = P\left(Z \le \frac{6.9-7.02}{0.08132}\right) \doteq P(Z \le -1.48) \doteq 0.0694$.

5.11. **(a)** With $n = 200$, the 95% probability range was about ± 10 minutes, so need a larger sample size. (Specifically, to halve the range, we need to roughly quadruple the sample size.) **(b)** We need $2\sigma_{\bar{x}} = \frac{5}{60}$, so $\sigma_{\bar{x}} \doteq 0.04167$. **(c)** With $\sigma = 1.15$, we have $\sqrt{n} = \frac{1.15}{0.04167} = 27.6$, so $n = 761.76$—use 762 students.

5.12. **(a)** The standard deviation is $\sigma/\sqrt{10} = 280/\sqrt{10} \doteq 88.5438$ seconds. **(b)** In order to have $\sigma/\sqrt{n} = 15$ seconds, we need $\sqrt{n} = \frac{280}{15}$, so $n \doteq 348.4$—use $n = 349$.

5.13. Mean $\mu = 250$ ml and standard deviation $\sigma/\sqrt{6} = 3/\sqrt{6} \doteq 1.2247$ ml.

5.14. **(a)** For this exercise, bear in mind that the actual distribution for a single song length is definitely *not* Normal; in particular, a Normal distribution with mean 350 seconds and standard deviation 280 seconds extends well below 0 seconds. The Normal curve for $\bar{x}$ should be taller by a factor of $\sqrt{10}$ and skinnier by a factor of $1/\sqrt{10}$ (although that technical detail will likely be lost on most students). **(b)** Using a $N(350, 280)$ distribution, $1 - P(331 < X < 369) \doteq 1 - P(-0.07 < Z < 0.07) \doteq 0.9442$. **(c)** Using a $N(350, 88.5438)$ distribution, $1 - P(331 < X < 369) \doteq 1 - P(-0.21 < Z < 0.21) \doteq 0.8336$.



5.16. For the population distribution (the number of friends of a randomly chosen individual), $\mu = 130$ and $\sigma = 85$ friends. **(a)** For the total number of friends for a sample of $n = 30$ users, the mean is $n\mu = 3900$ and the standard deviation is $\sigma\sqrt{n} \doteq 465.56$ friends. **(b)** For the mean number of friends, the mean is $\mu = 130$ and the standard deviation is $\sigma/\sqrt{n} \doteq 15.519$ friends. **(c)** $P(\bar{x} > 140) = P\left(Z > \frac{140-130}{15.519}\right) \doteq P(Z > 0.64) = 0.2611$ (software: 0.2597).