Stat 202
Fall 2014
Exam 2 Practice
10/30/14
Time Limit: 75 Minutes

Name (Print): _____

This exam contains 3 pages (including this cover page) and 12 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. A calculator is OK as long as it has no internet. You may use the browser on the lab computer (but not your computer) to access StatCrunch and to access my web page http://www.seancarver.org/. Access to my web page is granted for the sole purpose of downloading the data set that goes with this exam (exam1.xls under (data)). No other computer use is allowed.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

- If you need more space, use the back of the pages; clearly indicate when you have done this.

- Do not write in the table to the right.

Problem	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	15	
7	20	
8	15	
9	5	
10	5	
11	5	
12	25	
Total:	115	

Formulas that may or may not be useful:

- If X is a discrete random variable that takes on values $x_1, x_2, \ldots x_n$ with respective probabilities $p_1, p_2, \ldots, p_n$, then its mean is given by:

$$\mu_X = \sum x_i p_i$$

- The mean of a linear transformation of a random variable X is given by the following (where a and b are numbers, not random variables):

$$\mu_{a+bX} = a + b\mu_X$$

- The mean of a sum of two random variables X and Y is given by:

$$\mu_{X+Y} = \mu_X + \mu_Y$$

- A more general formula that combines the two above; here $X_1, \ldots, X_n$ are random variables and a and $b_1, \ldots, b_n$ are numbers:

$$\mu_{a+b_1 X_1 + b_2 X_2 + \ldots b_n X_n} = a + b_1 \mu_{X_1} + b_2 \mu_{X_2} + \ldots b_n \mu_{X_n}$$

- If X is a discrete random variable that takes on values $x_1, x_2, \ldots x_n$ with respective probabilities $p_1, p_2, \ldots, p_n$, then its variance is given by:

$$\sigma_X^2 = \sum (x_i - \mu)^2 p_i$$

- The variance of a linear transformation of a random variable X is given by the following (where a and b are numbers, not random variables):

$$\sigma_{a+bX}^2 = b^2 \sigma_X^2$$

- The mean of a sum of two independent random variables X and Y is given by:

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$$

- A more general formula that combines the two above; here $X_1, \ldots, X_n$ are independent random variables and a and $b_1, \ldots, b_n$ are numbers:

$$\sigma_{a+b_1 X_1 + b_2 X_2 + \ldots b_n X_n}^2 = b_1^2 \sigma_{X_1}^2 + b_2^2 \sigma_{X_2}^2 + \ldots b_n^2 \sigma_{X_n}^2$$

- Central limit theorem: For a population with mean μ and standard deviation σ, and for samples chosen of size n, the distribution of the sample mean is approximately normal:

$$\bar{x}_n \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

1. (5 points) Load the data "statcourse8," which lists the first test score and second test score for a hypothetical statistics class.

    (a) (5 points) Which variable should be consider the response variable and which variable should be considered the explanatory variable?

    (b) (5 points) What is the correlation between the two variables?

    (c) (5 points) Let s_1 be the first test score and s_2 be the second test score. Write the equation for the regression line. (The response variable and explanatory variables should be chosen appropriately.)

2. (5 points) What is the correlation between random variables X and Y where the two are related by $Y = 10 - 2X$?

3. (5 points) Suppose random variables X, Y and Z are related by the formula

$$Z = 3X + 4Y + 5.$$

   Suppose $\mu_X = 1$, $\mu_Y = 2$, $\sigma_X = 4$, and $\sigma_Y = 3$. Find μ_Z and σ_Z.

4. (5 points) Suppose the population mean for people who work in a particular building is 175 lbs, and suppose the population standard deviation is 10 lbs. Suppose three random people get on an elevator and one is pushing a cart that weighs 50 lbs. Let Z be the load on the elevator (i.e. the weight of what it is carrying). Describe Z in terms of its mean and standard deviation.

5. (5 points) Suppose the population mean for people who work in a particular building is 175 lbs, and suppose the population standard deviation is 10 lbs. The elevator has a maximum capacity of 10 people. Let Z be the load with 10 random people. Let W be the average weight of the ten people on the elevator. Find μ_Z, Find σ_Z, Find μ_W, Find σ_W.

6. (15 points) Suppose distribution of test scores on a standardized test is Normal with mean 515 and standard deviation 41.

    (a) (5 points) Find the z-score for a student with score 450.

    (b) (5 points) On what percentile does a score of 450 fall?

    (c) (5 points) What score is required to fall on the 95th percentile?

7. (10 points) A casino is considering a game where a 20 sided dice is thrown 20 times. If it turns out that a 2 6 10 or 12 appears 5 or more times the player wins, otherwise the house wins.

   (a) (5 points) What is the probability that the house wins?

   (b) (5 points) If the tickets cost $10, what is the largest payoff the casino can afford to give the player for winning and still expect to break even?

8. (15 points) Lead has been added the 6-face of a six-sided die so that 6 appears with probability 0.21. Suppose the probabilites of 2,3,4, and 5 are not affected, but the probability of a 1 is affected.

   (a) (5 points) What is the probability of rolling a 1?

   (b) (5 points) What is the probability of rolling a 2?

   (c) (5 points) Let X be the random variable equal to the number of dots shown on the trown die. What is the mean and standard deviation of X.

   (d) (5 points) Suppose there are two such dice, and they are identical. What is the mean and standard deviation of the sum of the dots on the two dice, both thrown together?

9. (5 points) To use an Automatic Teller Machine (ATM) you must type in your Personal Identification Number (PIN), a four digit number that is sometimes selected by the user and sometimes assigned randomly. What is the probability that the four digit PIN contains a zero? Explain your reasoning.

10. (5 points) What is the probability that 5 fair dice rolled together contains a 6? (Each side of a fair die has an equal probablity of showing up when thrown). Explain your reasoning.

11. (5 points) Government data show that 6% of the American population are at least 75 years of age and about 51% are women. Explain why it is wrong to conclude that because (0.06)(0.51) = 0.0306, 3% of the population are women aged 75 or over.

12. (25 points) The heights of people in our population of 1000 individuals have mean 66 inches and standard deviation 4 inches. A random sample of 36 is chosen, and the average (mean) height of the 36 people is calculated.

   (a) (5 points) If samples of 36 were drawn repeatedly and for each the average (sample) height was computed, what is the mean of this average?

   (b) (5 points) What is the standard deviation of this average?

   (c) (5 points) Use the 68-95-99.7 rule to calculate an interval in which 95% of the sample averages fall. What is the lower and upper endpoints of this interval?

   (d) (5 points) What is the probability that the mean will be above 66.1?

   (e) (5 points) If you want 95% of your sample means to fall between 65.9 and 66.1, what should your sample size be?